ESTIMATION OF MILK PRODUCTION FUNCTIONS WITH RESPECT TO RATE OF GRAIN FEEDING, STAGE OF LACTATION AND PRODUCING ABILITY

By

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INTRODUCTION

Since feed is usually considered the largest single item of cost in milk production, the optimum ration is of great economic importance to the dairyman. If information were available to the dairyman on the amount of milk to expect from a given input of roughage and concentrates, he would be better qualified to choose a profitable feeding program.

With a milk production function which could be used to estimate milk yield under certain conditions, physical input-output relationships could be predicted. Derivation of such physical quantities would allow specification of the most profitable feeding plan, given the cost of feed and the price of milk.

The purpose of this study was to secure data which when reduced to a prediction equation could be used to predict milk production under certain conditions with respect to rate of grain feeding, stage of lactation and producing ability of the cow.

The primary objective of this investigation was methodological in nature, involving means of estimating milk production functions rather than the use of these particular principles in determining economic optima in dairy feeding.

REVIEW OF LITERATURE

Numerous investigations (2, 3, 8, 10, 11, 12, 13, 15, 20, 23, 24, 25, 26, 31, 32, 35) have shown that generally milk may be produced more economically from a low-grain-high-roughage ration. However, since there is fluctuation in the costs of concentrates and roughages and in the price of milk, conditions may be such that a high-grain-low-roughage ration would be the most profitable feeding plan. The point at which the greatest net return can be realized is when the cost of the last unit of input just equals the value of the additional milk produced.

The postulation that milk production increases at a diminishing rate as feed intake is increased has been shown to be true in several investigations. Jensen *et al*. (18) found that milk production increased as TDN consumption increased but at a diminishing rate, and that it was not possible to feed the cow enough to reach a point where an increase in milk production would no longer result. Yates *et al*. (36) summarized several research reports implying a diminishing rate of transformation of feed into milk. Ashe (1) obtained data from a sample of dairy farms which showed that input-output curves followed a near linear relationship up to about 4,000 lbs. of grain per cow; between 4,000 and 6,000 lbs. of grain, milk increased only slightly; and over 6,000 lbs. of grain, milk did not increase at all. However, the

Ashe data are based on farm surveys and it is likely that inherent cow ability and management effects are confounded with ration effects.

The daily recommended allowance for each additional pound of 4% fat corrected milk above maintenance is 0.32 lb. TDN, as prescribed by the Committee on Animal Nutrition of the National Research Council (7) and by Morrison (24). These recommendations do not take into account the concept of diminishing returns as it pertains to dairy cow rations.

A number of studies (2, 8, 9, 10, 12, 20, 23, 25, 26, 31, 32) have been conducted in which various rules of thumb have been used to allot concentrates to lactating dairy cows. However, there are considerable differences of opinion among these workers in regard to the most profitable rate of grain feeding.

Woodward (33) contends that the commonly used method of apportioning grain to cows by allowing 1.0 lb. of grain for a certain number of pounds of milk produced tends to over-feed low-producing cows and underfeed high-producing cows. The opinion of these workers was that the most practical method of allotting grain to cows was to allow high-butterfat cows 0.6 lb. of grain per 1.0 lb. of milk produced daily above 10 lbs. and feed low-butterfat cows 0.4 lb. of grain per 1.0 lb. of milk produced above 16 lbs., assuming the cows had access to all the good quality roughage they would want. However, it is obvious that rules based on such ratios ignore price-cost relationships.

Porter and Blake (28) reported that there was no significant difference between the milk production of two groups of cows, one in which grain was fed individually and another in which grain was group-fed.

The same total amounts were fed to both groups. The authors concluded that the loose-feeding method was more profitable than individual feeding considering differences in the labor involved.

Lewis (21) conducted a study to find how closely related a statement of average concentrate-milk ratio in a dairy herd would be to the actual feeding rates of the individual cows in the herd when the cows were fed according to level of milk production, fat test of the milk, stage of lactation, condition of the cow and the type and quality of roughage fed. The data for this study were taken from one year's grain feeding and milk production records of a well-managed herd. The findings supported the worker's contention that as long as these factors are considered in allotting feed to individual animals, no common feeding rate will be found.

Bloom et al. (4) conducted a study in which thirty-six cows were divided into high-, medium- and low-producing groups in order to evaluate their efficiency for milk production. Four different hay-concentrate energy ratios were studied at high, medium and low levels of intake. One cow from each of the three producing-ability groups was assigned to each of the hay-concentrate feeding level groups. It was found that regardless of the ratio of concentrates to hay fed, the inherent ability of the cows to produce milk was more significant than the intensity of feeding.

Jensen et al. (18) found that there was considerable difference in the response in milk production of cows of low inherent productivity and of cows of high inherent productivity to increased grain allowances. The response to increased feeding was not as great with cows of low inherited productivity as it was with cows of high inherited productivity. However, the high and low cows responded about the same in terms of percentages of the previous producing rate before the increase in feeding was made.

Jensen et al. (18), in cooperation with ten Experiment Stations, obtained 469 yearly records of milk production and feed consumption from 346 individual cows. To show the degree of increase in production from an increase in feeding, input-output curves were constructed by fitting the data with the Spillman function:

$Y = M - AR^X$

Y = Milk production

M = Theoretical maximum production

A = Difference between Y at $X = 0$ and Y at M

R = A geometric ratio between successive increments of Y

X = Input of grain

These curves were non-linear, indicating that the increase in yield was not the same for each added quantity of feed. These curves were used as the basis for making a feeding chart which would show the most profitable feeding plan under given prices of roughage, grain and milk.

Heady et al. (16) obtained data from the study made by Bloom et al. (4) in which 36 Holstein cows were divided into three producing-ability groups and fed different levels and combinations of concentrates and roughage. All the cows got the same ration for a 60 day preliminary period and then each group received only one of the test rations during the experimental period. From these data milk production functions were formulated which could be used to predict the milk production surface under particular conditions. The variables considered in the model equation were grain consumption, hay consumption, time point in the lactation period and producing ability of the cow. The model, which estimates milk production for a four-week period, may be illustrated by the following equation in which M represents milk yield, H = hay consumption, G = grain consumption, T = a four-week time period during the lactation and A = the milk producing ability of the cow measured as the amount produced during a preliminary period:

$$M = 1.6302H + 3.1309G + 0.1497A + 14.2243T - 0.000388H^2 - 0.001192G^2 + 4.3792T^2 - 0.00105HG - 0.175GT - 0.0865HT - 731.76$$

Here, the regression equation explains 81.3% of the variation in production over the experimental period. The relationship was also as the following logarithmic equation where 74.9% of the variation was explained by regression:

$$M = 4.1937H^{.1506}G^{.3082}A^{.3716}T^{-.1973}$$

Cochran et al. (5) claim that response in milk production from a given ration would not be influenced by variations in producing ability of the cows if each cow received each ration during the experimental period. However, when using a short-time switch-over design, a carry-over effect of the ration given in the previous period may be anticipated. A procedure was described by which the short-time switch-over or Latin square design could be used in dairy cow feeding trials in such a way that adjustments could be made for carry-over effects and accurate and unbiased comparisons of the effects of the rations could be obtained.

EXPERIMENTAL PROCEDURE

Twelve purebred Holstein cows were selected from the Oklahoma State University dairy herd for this study. Data collected were used to estimate equations that would predict milk production in relation to level of grain feeding, stage of lactation and producing ability. Three levels of grain feeding were used while roughage was fed free-choice.

One cow had to be removed from the experiment because of sickness, however, the design of the study was such that the analysis of the data obtained from the remaining eleven cows was not confounded.

Design of the Experiment

This experiment consisted of two 90-day trials with each trial being divided into three 30-day periods. Each cow was subjected for 30 days to each of three levels of grain feeding in a manner described by the Latin square experimental design. This design was used so that the effects of variable producing ability would be minimized. After having completed the first 90-day trial, the cows were re-randomized into new groups and fed at the three levels for the second 90 days as before although not in the same sequence.

Feeding and Management

The cows had continual access to corn silage in a self-feeding trench silo and alfalfa hay in conventional racks. The three levels of grain feeding were as follows: high, which was 110% of Morrison's recommended grain allowance; medium, which was 62.5% of the high rate; and low, which was 62.5% of the medium rate.

A concentrate mixture of the following composition was fed: ground milo, 800 lbs.; ground oats, 600 lbs.; wheat bran, 600 lbs.; and salt and dicalcium phosphate, 20 lbs. each.

Grain was allotted to the cows in measured amounts twice daily as they were being milked. Weekly adjustments in the allowances of grain were made in accordance with the previous week's average rate of milk production.

The stage of lactation at which the cows were removed from normal herd conditions and placed on trial ranged from two to eight weeks after calving. All the cows with the exception of two began the trial at the same time.

The cows were allowed to run outside at all times except for two periods daily of approximately five minutes each during which time they were milked in a milking parlor.

Statistical Procedure

A production function is a means of describing an input-output relationship, the amount of output being dependent upon the quantity and quality of the input. Milk production is a complex process which is influenced by many factors (feed, stage of lactation, producing ability, management and others); therefore, as the combination and level of these factors vary, production will also vary. A production function representing milk production may be expressed as follows:

$Y = f(X_1, X_2, X_3/X_4 \ldots\ldots X_n)$

Y = Milk production

X_1 = Grain intake

X_2 = Time period during the lactation

X_3 = Producing ability

$X_4 \ldots\ldots X_n$ = Roughage intake, management and other relevant factors

The vertical line between X_3 and X_4 indicates that all inputs to the right of the line are considered fixed while those to the left are variable. The equation states that Y depends upon the application of X_1, X_2, X_3, $X_4 \ldots\ldots X_n$, and a change in the combination and level of the independent variables (X_1, X_2, X_3, $X_4 \ldots\ldots X_n$) will result in a change of output (Y).

For the purpose of statistical treatment of the data, the two 90-day trials of this experiment were divided into 10-day periods and the milk production and grain consumption per 10-day period were totaled. This arrangement of the data is shown in tabular form in the appendix.

First, a function was estimated from the data which contained two variables, rate of grain feeding and time period during the lactation. Only data obtained during the first 90-day trial were used in estimating this function. The following equation was fitted to the data:

$$Y = aX_1^{b_1} X_2^{b_2}$$

Here, Y refers to milk production in a 10-day period, X_1 refers to grain intake for the 10-day period, and X_2 refers to a 10-day time period during the lactation.

Production functions were derived from the experimental data obtained during the entire 180-day trial which contained a third variable, producing ability. The producing ability of the cow was measured in pounds of 4% fat corrected milk produced during a 10-day preliminary period prior to the experiment. The equation, now containing three variables, was used to describe three milk production functions formulated from the data which will hereafter be referred to as functions (1), (2) and (3).

Function (1) was derived from data obtained using all 18 of the 10-day periods.

Function (2) was determined by omitting the first ten days spent on each level of grain feeding and fitting the equation to data secured only from the last 20 days of each of the 30-day periods.

Function (3) was obtained by using data from only the third 10-day period of each of the 30-day periods.

Functions (2) and (3) were fitted to see if carry-over effects were influencing the milk production response on given rates of feeding. It would be expected that, normally, the response in milk from a given rate of grain feeding would be greater if the cow had been fed at a higher rate during a previous period than if she had been fed at the same rate during the preceding period. A reverse reaction would be expected if the cow were fed at a lower rate during the previous period. By omitting the first 10 or 20 days' data in each 30-day period from the analysis, carry-over effects would be eliminated only if the production during the last 20 or 10 days was identical to the production if the cow had been on the same rate of feeding for all previous periods.

RESULTS AND DISCUSSION

An equation which involved two variables, grain intake and time, was fitted to data available from the first 90-day period and the following equation was obtained:

$$Y = 158.3X_1^{.2237}X_2^{-.0729} \qquad R^2 = .3347$$

The function explained 33.47% of the variance in milk production.

Because of this small R^2 value a third variable, producing ability, was included in the equation that was fitted to the 180-day data.

Three functions which were derived from analysis of the 180-day data and the related statistics are shown in Table 1.

The larger R^2 values for the functions which included the third variable indicated that this third factor, producing ability, accounted for a large part of the variation in milk production in the experimental data.

The R^2 values for the functions determined in this study were slightly smaller than those for the functions obtained in the study made by Heady et al. (16). The reason for this difference can probably be explained by the fact that hay consumption was a controlled variable in the Heady study, while in this study hay intake was not controlled or measured.

The differences in the R^2 values of function (1) in which all the 180-day data were used in the analysis, function (2) in which the first 10 days of each 30-day period were omitted, and function (3) in which

TABLE 1

Regression Coefficients and Related Statistics of Three Functions of the Basic Equation, $Y = aX_1^{b1} X_2^{b2} X_3^{b3}$

Function	"a" value		b_i	t_{b_i}	R^2	F
(1)	5.106 (log. .7081)	b_3	0.5616**	9.7542	.6572	92.99
		b^2	-0.1075**	6.6560		
		b^1	0.2202**	8.2422		
(2)	5.096 (log. .7072)	b^3	0.5733**	8.1995	.6665	63.94
		b^2	-0.1316**	5.7867		
		b^1	0.2153**	6.8299		
(3)	4.524 (log. .6555)	b^3	0.5898**	6.0180	.6807	37.07
		b^2	-0.1380**	3.8862		
		b^1	0.2220**	5.0410		

**Significant at the .01 level.

the first 20 days of each 30-day period were omitted, indicate that possibly carry-over effects from a previously fed ration had some influence on the response in milk production for given levels of grain feeding.

With equations of this type for predicting milk production, milk yield for a time period may be estimated by substituting for X_1, the grain input for the period, for X_2, the time period during the lactation, and for X_3, the amount of milk produced during a preliminary period at the peak of lactation. In the functions developed in this study, the length of the time periods is ten days. The first 10-day period, for practical purposes, may be considered to begin just after the peak of lactation is reached or sometime between 30 and 40 days after freshening since the average stage of lactation at which the experimental cows were placed on trial was five weeks after calving. These functions are applicable to situations where roughage is fed free-choice.

In Figures 1 and 2 milk production functions are presented as response curves. Function (3) was used in all instances since this equation seemed to fit the experimental data better than the other two.

In Figure 1 the upward sloping curves illustrate the functional relationships between grain consumption and milk production at four producing-ability levels during the first 10-day period after the peak of lactation is reached.

The response curves demonstrate the concept of diminishing returns which means that the rate of grain transformation into milk decreases as the input of grain increases. Points have been found on the curves

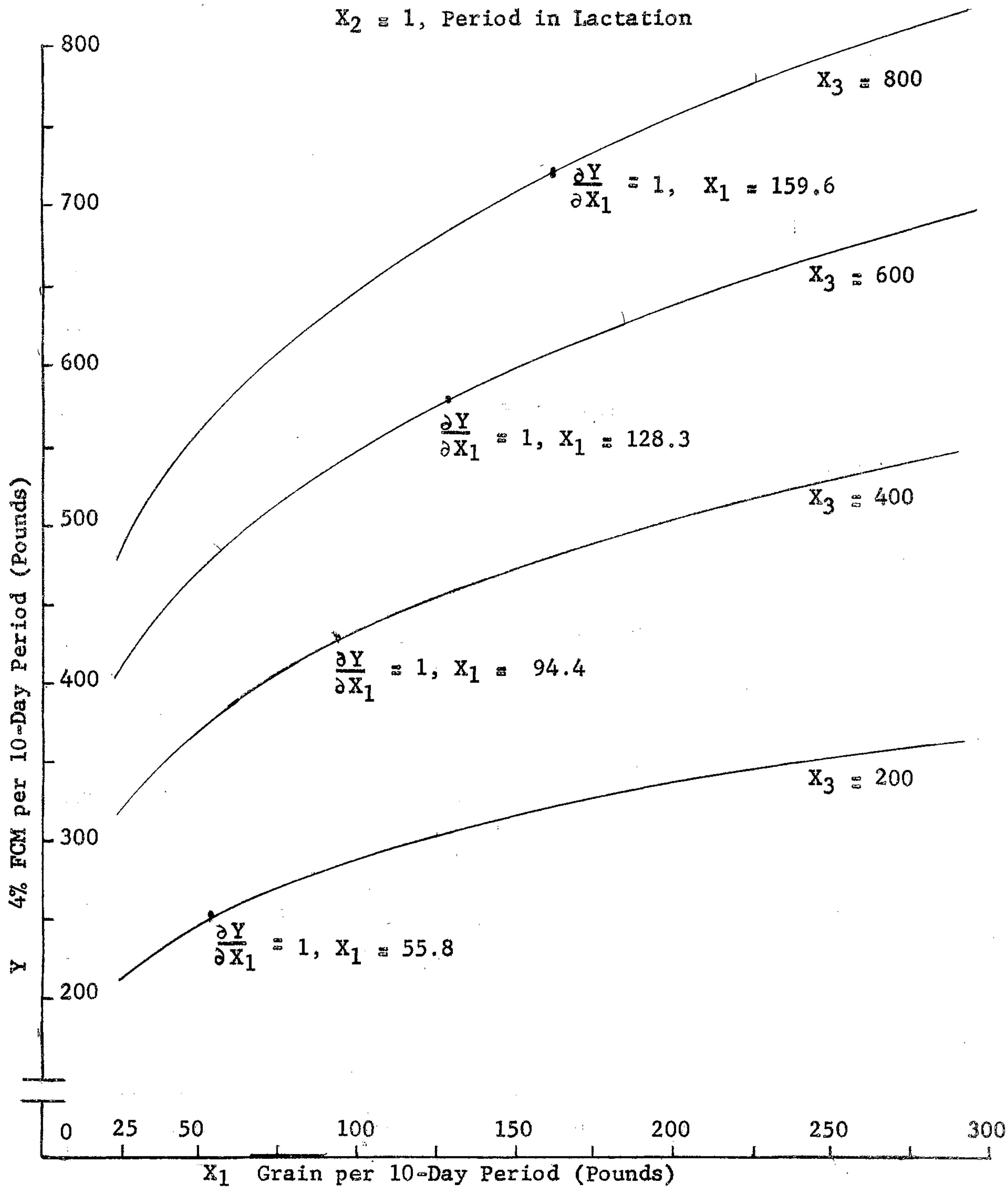

Figure 1. Response Curves Showing Transformation Rate of Grain to Milk for Four Producing-Ability Levels During the First 10-Day Period After the Peak of Lactation.

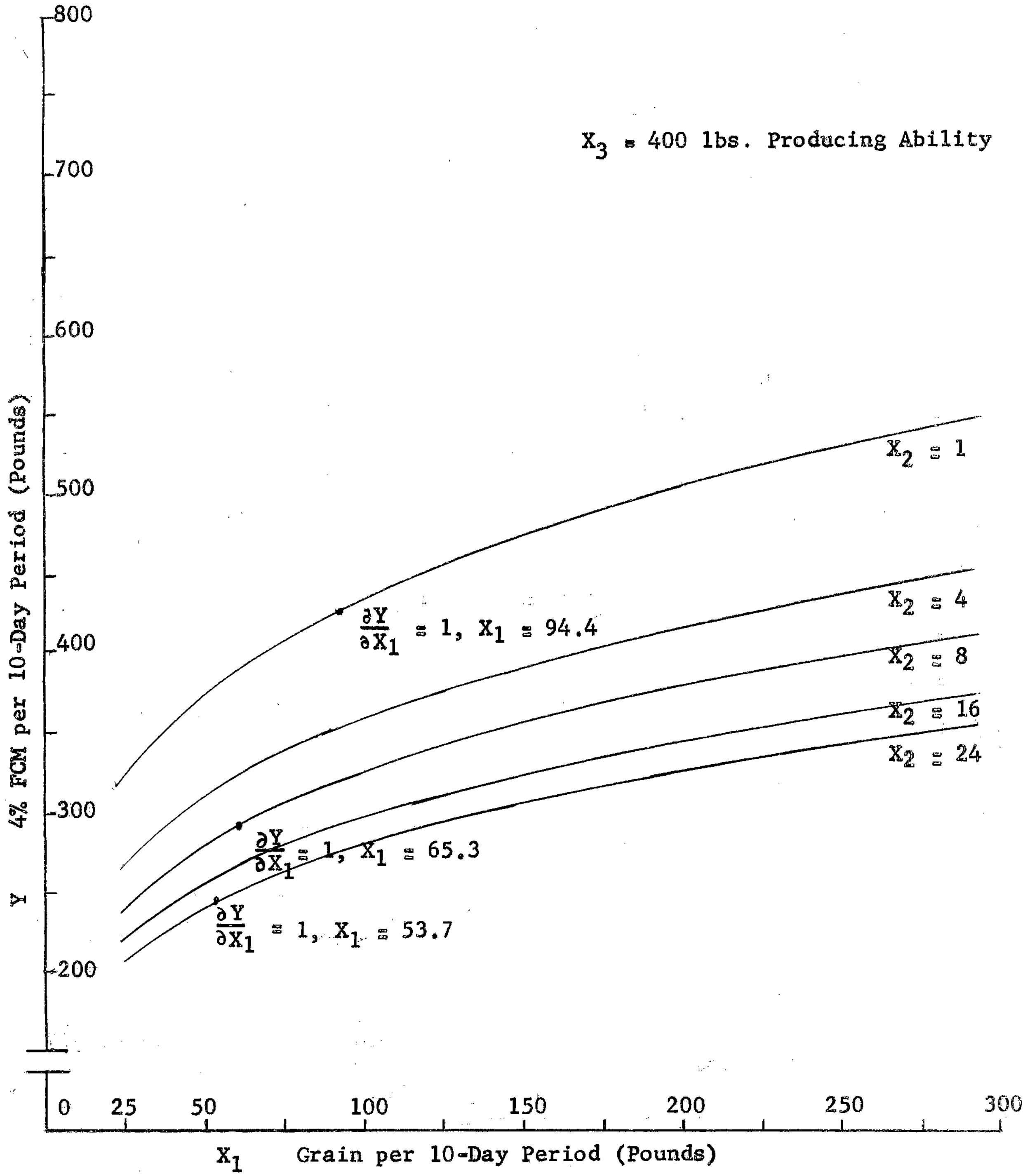

Figure 2. Response Curves Showing Transformation of Grain to Milk for Five Time Periods During the Lactation at One Producting-Ability Level

where the change in milk output (∂Y) is equal to the change in grain input (∂X_1). At any place on the curves below the point $\partial Y/\partial X_1 = 1$ the response in units of milk is greater than the units of grain required to produce the response. At any place on the curves above $\partial Y/\partial X_1 = 1$ the unit response of output is less than the required units of input.

A comparison of the response curves in Figure 1 shows that the curves for the lower producing cow level off more rapidly than those for the higher producing cows, demonstrating that low producing cows tend to reach the point where the grain-milk transformation ratio is one at lower levels of grain feeding than do higher producing cows.

In Figure 2 the functional relationships between feed intake and milk output are shown for five periods during the lactation at one producing-ability level. The slope of the transformation curves is greater for the early lactation periods than for the later periods, demonstrating that the rate of transformation of grain to milk as grain input increases does not diminish as fast during the first part of the lactation as during later stages.

The optimum level of grain feeding is when the value of the added milk just equals the value of the added grain and may be expressed by the following equation:

$$\frac{\partial M}{\partial G} = \frac{Pg}{Pm}$$

$\partial M/\partial G$ is the marginal product, the amount added to total milk production by an increase of one unit of grain, Pg is the price per unit of grain, and Pm is the price per unit of milk. If the price of milk and grain are equal then the most profitable level of grain feeding would be when $\partial M/\partial G = 1$. If the marginal product is less than the grain-milk price ratio, $\partial M/\partial G < Pg/Pm$, profit can be increased by feeding less grain since the value of the milk sacrificed is less than the value of the grain subtracted. If the marginal product is greater than the price ratio, $\partial M/\partial G > Pg/Pm$, higher grain levels will add more to the returns from the increase in milk yield than to the cost of the feed added.

When an appropriate milk production function is known, the partial derivative, $\partial M/\partial G$, may be obtained and equated to the grain-milk price ratio:

$$M = 4.524G^{.222}T^{-.138}A^{.5898}$$

$$\frac{\partial M}{\partial G} = 1.004G^{-.778}T^{-.138}A^{.5898}$$

$$\frac{\partial M}{\partial G} = \frac{Pg}{Pm}$$

$$\frac{Pg}{Pm} = 1.004G^{-.778}T^{-.138}A^{.5898}$$

By solving for G the economic optimum level of grain feeding may be determined.

By this process values may be obtained that could be used in the compilation of tables such as Table 2 which could be useful to dairymen in maximizing income above feed costs. It should be pointed out that any milk production function may not fit all situations. For instance, values derived from the function obtained in this study would be more useful if applied under conditions where the type, quality and method of feeding the roughage were similar to those used in this experiment. The same may be said relative to inferences from any experiment.

TABLE 2

Example of Daily Recommended Grain Allowances for Cows Allotted Roughage Free-Choice, at Three Producing-Ability Levels, for the First 10-Day Period After the Peak of Lactation, Based on a Function Predicted from a 12-cow Experiment, 1956-57. (Pounds)

Price of Grain per Cwt.	Price of Milk Per Hundred Pounds																	
	$3.50			$4.00			$4.50			$5.00			$5.50			$6.00		
	Producing Ability at Peak of Lactation (Pounds per Day)																	
	30	50	70	30	50	70	30	50	70	30	50	70	30	50	70	30	50	70
$2.00	13.3	19.6	25.3	15.2	22.4	28.9	17.1	25.2	32.5	19.0	27.9	36.1	20.9	30.7	39.6	22.8	33.5	43.3
$2.50	10.6	15.7	20.2	12.1	17.9	23.1	13.7	20.1	26.0	15.2	22.4	28.9	16.7	24.6	31.7	18.2	26.8	34.6
$3.00	8.9	13.0	16.8	10.1	14.9	19.2	11.4	16.8	21.7	12.7	18.6	24.0	13.9	20.5	26.5	15.2	22.4	28.9
$3.50	7.6	11.2	14.4	8.7	12.8	16.5	9.8	14.4	18.6	10.8	16.0	20.6	11.9	17.6	22.7	13.0	19.2	24.7
$4.00	6.6	9.8	12.6	7.6	11.2	14.4	8.5	12.6	16.3	9.5	14.0	18.0	10.4	15.4	19.9	11.4	16.8	21.7
$4.50	5.9	8.7	11.2	6.8	9.9	12.8	7.6	11.2	14.4	8.4	12.4	16.0	9.4	13.8	17.8	10.1	14.9	19.2

Table 2 shows the most profitable level of grain feeding for various grain-milk price ratios at three levels of producing ability when roughage is fed free-choice. The table applies to the first 10-day period after the peak of lactation is attained; however, it could be expanded or supplemented with a conversion formula so that it could be applicable to other periods.

Table 2 was prepared to serve as an illustration of one of the practical economic aspects of refined milk production-prediction equations of this type. Beside having other economic potentials, production functions may be useful to nutritionists and others who are concerned with predicting the outcome of rations.

SUMMARY AND CONCLUSIONS

The milk production of eleven Holstein cows, divided into three groups, was observed when roughage was fed free-choice and grain at three levels. A Latin square experimental design was used so that each group spent 30 days on each level of grain feeding. After completing the first 90-day trial the cows were regrouped and subjected to a similar trial.

By fitting a regression equation to the data obtained during the first 90 days the following milk production function was derived:

$$Y = 153.8X_1^{.2237}X_2^{-.0729}$$

Y = Milk yield for a 10-day period

X_1 = Grain intake for a 10-day period

X_2 = Time period during the lactation

A third variable, producing ability, was included in the equation which was fitted to the 180-day data. Milk production during the 10 days prior to the trial was used as a measure of the producing ability. The following functions were obtained:

$$(1)\quad Y = 5.106X_1^{.2202}X_2^{-.1075}X_3^{.5616} \qquad R^2 = .6572$$

$$(2)\quad Y = 5.096X_1^{.2153}X_2^{-.1316}X_3^{.5733} \qquad R^2 = .6665$$

$$(3)\quad Y = 4.524X_1^{.2220}X_2^{-.1380}X_3^{.5898} \qquad R^2 = .6807$$

Function (1) involved all of the 180-day data in the analysis, function (2), only the last 20 days of each 30-day period, and function (3), only the last 10 days of each 30-day period.

Function (3) was selected as the best fitting equation because it had the highest R^2. It showed that milk output increased at a diminishing rate as grain input increased, and that this rate of transformation was influenced by the stage of lactation and the producing ability of the cow. The function was used to predict the optimum level of grain feeding for various grain-milk price ratios.

It is not suggested that the design of this study was ideal nor that the values obtained are final. However, the results possess economic and nutritional logic that should encourage further work that will allow more refined predictions of milk production functions. It is hoped that this study will provide the basis for and prove useful in designing other investigations of this type.

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A P P E N D I X

APPENDIX

TABLE I

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 1

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	477.7	-
	1	170	467.9	
High	2	176	515.0	
	3	198	483.2	1273
	4	70	424.9	
Low	5	64	362.2	
	6	51	345.1	1317
	7	77	352.8	
Medium	8	70	331.9	
	9	70	340.1	1317
	10	123	410.8	
High	11	130	400.9	
	12	121	391.1	1318
	13	40	346.3	
Low	14	40	340.1	
	15	40	322.6	1267
	16	60	301.5	
Medium	17	66	300.0	
	18	70	307.0	1237

TABLE II

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 2

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	523.4	-
	1	230	531.3	
High	2	242	543.0	
	3	241	541.8	1363
	4	100	570.6	
Low	5	94	520.7	
	6	88	451.0	1410
	7	110	473.9	
Medium	8	104	445.8	
	9	100	442.5	1434
	10	90	382.0	
Medium	11	90	372.8	
	12	90	379.6	1384
	13	159	440.2	
High	14	180	440.0	
	15	198	452.8	1404
	16	80	412.4	
Low	17	74	354.8	
	18	61	355.2	1329

TABLE III

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 3

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	584.0	-
	1	230	573.8	
High	2	224	487.6	
	3	184	493.5	1313
	4	77	504.8	
Low	5	64	471.1	
	6	60	430.9	1320
	7	84	381.8	
Medium	8	70	380.0	
	9	70	376.3	1350
	10	50	362.6	
Low	11	50	354.9	
	12	48	267.7	1329
	13	60	274.3	
Medium	14	60	260.3	
	15	60	261.7	1342
	16	90	268.1	
High	17	84	242.0	
	18	71	242.1	1360

TABLE IV

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 4

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	580.0	-
	1	171	464.8	
High	2	192	579.6	
	3	211	583.8	1325
	4	80	485.0	
Low	5	74	462.3	
	6	61	457.3	1325
	7	110	459.8	
Medium	8	110	467.2	
	9	101	435.3	1340
	10	172	486.7	
High	11	218	508.0	
	12	221	491.8	1317
	13	80	480.2	
Low	14	80	481.5	
	15	82	484.6	1278
	16	150	557.4	
Medium	17	144	543.8	
	18	140	551.4	1268

TABLE V

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 5

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	428.3	-
	1	100	380.0	
Medium	2	100	406.1	
	3	100	388.9	1378
	4	150	392.6	
High	5	162	396.5	
	6	161	384.2	1387
	7	60	425.2	
Low	8	54	406.1	
	9	50	382.2	1379
	10	50	331.8	
Low	11	50	315.3	
	12	50	305.4	1361
	13	80	315.2	
Medium	14	80	312.8	
	15	80	300.4	1340
	16	123	321.2	
High	17	130	310.2	
	18	121	315.3	1296

TABLE VI

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 6

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	268.1	-
	1	67	258.3	
Medium	2	60	272.1	
	3	69	277.8	1448
	4	110	328.3	
High	5	116	339.3	
	6	118	315.4	1470
	7	40	289.1	
Low	8	40	274.7	
	9	40	260.6	1473
	10	60	275.0	
Medium	11	60	259.2	
	12	51	238.3	1481
	13	90	241.8	
High	14	90	229.7	
	15	90	221.9	1450
	16	30	214.2	
Low	17	30	203.7	
	18	30	185.5	1398

TABLE VII

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 7

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	520.9	-
	1	124	375.0	
Medium	2	110	391.6	
	3	119	369.4	1411
	4	180	435.3	
High	5	180	302.3	
	6	156	351.2	1470
	7	50	396.9	
Low	8	50	406.8	
	9	50	389.9	1437
	10	47	333.3	
Low	11	40	331.4	
	12	40	321.3	1431
	13	73	317.0	
Medium	14	80	315.7	
	15	80	327.2	1414
	16	120	332.1	
High	17	120	339.1	
	18	120	332.1	1424

TABLE VIII

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 8

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	350.0	-
	1	64	299.5	
Low	2	56	281.7	
	3	51	266.0	1318
	4	87	341.6	
Medium	5	74	335.4	
	6	70	305.2	1333
	7	100	345.2	
High	8	94	307.1	
	9	90	303.3	1368
	10	80	265.8	
High	11	80	244.0	
	12	80	229.0	1390
	13	30	216.5	
Low	14	30	205.4	
	15	30	204.3	1359
	16	40	201.7	
Medium	17	40	208.7	
	18	40	207.4	1377

TABLE IX

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 9

Level of Conc.	10-Day Time Per During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	450.1	-
	1	67	384.1	
Low	2	60	352.4	
	3	60	333.5	1453
	4	90	354.9	
Medium	5	84	341.8	
	6	80	334.7	1479
	7	120	348.1	
High	8	120	351.7	
	9	120	321.4	1459
	10	40	301.0	
Low	11	34	268.3	
	12	30	254.2	1488
	13	50	247.4	
Medium	14	50	241.7	
	15	59	241.3	1462
	16	87	228.0	
High	17	80	219.6	
	18	71	212.6	1471

TABLE X

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 10

Level of Conc.	10-Day Time Per. During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	427.8	-
	1	77	319.4	
Low	2	58	269.6	
	3	52	270.8	1424
	4	77	295.3	
Medium	5	64	281.0	
	6	67	254.7	1466
	7	80	270.3	
High	8	86	274.2	
	9	81	245.5	1497
	10	70	242.1	
High	11	70	222.0	
	12	61	215.6	1496
	13	30	219.7	
Low	14	30	220.2	
	15	30	212.0	1481
	16	40	200.8	
Medium	17	40	198.6	
	18	40	207.2	1461

TABLE XI

FEED CONSUMPTION AND MILK PRODUCTION PER 10-DAY PERIOD AND BODYWEIGHT PER 30-DAY PERIOD, COW NO. 11

Level of Conc.	10-Day Time Per During Expt.	Conc.	4% FCM	Bodyweight
		lb.	lb.	lb.
-	Preliminary	-	475.0	-
	1	80	484.6	
Low	2	74	464.4	
	3	79	469.2	1332
	4	107	417.4	
Medium	5	88	375.7	
	6	89	334.6	1399
	7	100	347.7	
High	8	106	354.5	
	9	101	288.8	1437
	10	60	277.1	
Medium	11	60	269.9	
	12	60	276.5	1435
	13	100	273.3	
High	14	94	270.2	
	15	90	244.0	1451
	16	37	232.5	
Low	17	30	247.9	
	18	30	279.9	1413

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